Gravitational Anomaly and Transport

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E. Megías et al. Phys. Rev. Lett. 107:021601(2011), JHEP 1109:121(2011), arXiv:1110.3615[hep-ph] and arXiv:1111.2823[hep-th].

Issues

1. Anomalous Effects and Kubo Formulas
   - The Chiral Magnetic Effect
   - Kubo Formulas
2. Hydrodynamics of Relativistic Fluids
   - Weak coupling
   - Strong coupling
   - Fluid/Gravity Correspondence

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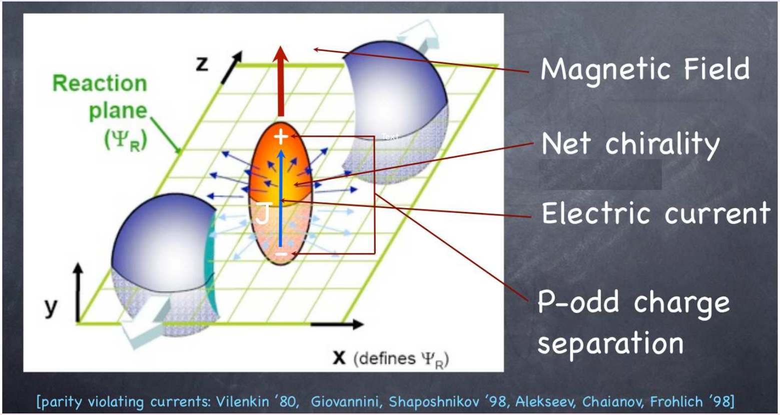
Issues

The Chiral Magnetic Effect

[Kharzeev, McLerran, Warringa '07]



Strong Magnetic field induces a P-odd charge separation $\Longrightarrow$

$\Longrightarrow$ Electric current: $\vec{J} = \sigma^{\mathcal{B}} \vec{B}$.

Issues

Kubo Formulas

- Chiral Magnectic Conductivity **[Kharzeev, Warringa '09]**

$$\vec{J} = \sigma^{\mathcal{B}} \vec{B} \,,$$

$$\stackrel{\text{In momentum space}}{\Longrightarrow} \qquad J_i = \sigma^{\mathcal{B}} \epsilon_{ijk} i p_j A_k \,.$$

- Kubo formula for a general symmetry group **(A. Gynther et al., JHEP1102(2011)110.)**

$$[T_A, T_B] = i f_{AB}{}^{C} T_C \,,$$

$$(\sigma^{\mathcal{B}})_{AB} = \lim_{p_j \to 0} \sum_{i,k} \frac{i}{2p_j} \epsilon_{ijk} \langle J_A^i J_B^k \rangle \Big|_{\omega=0} \,.$$

Retarded correlation function of currents.

Kubo Formulas

- Chiral fermions

$$J_i^A = \sum_{f,g=1}^{N} (T^A)^g{}_f \bar{\Psi}_g \gamma_i \mathcal{P}_+ \Psi^f \,, \qquad \mathcal{P}_+ = \frac{1}{2}(1+\gamma_5)\,,$$

- Chemical potentials and generators in the Cartan subalgebra

$$\mu^f = \sum_A q_A^f \mu_A \,, \qquad H_A = q_A^f \delta^f{}_g \,.$$

Chemical potentials break group $G \to \hat{G}$ fulfilling $T_A{}^f{}_g \mu^g = \mu^f T_A{}^f{}_g$. Only currents that lie in $\hat{G}$ are conserved and participate in the hydrodynamics.

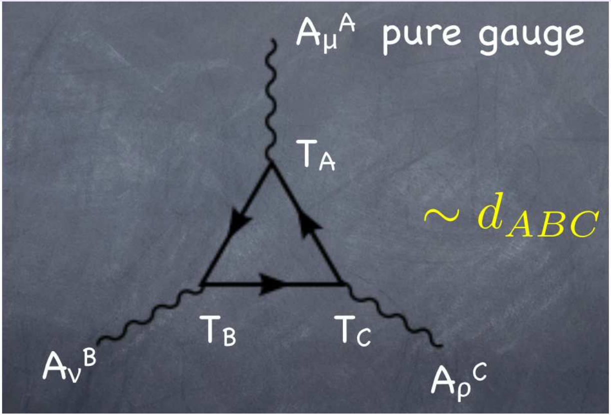
- 1-loop calculation [Kharzeev, Warringa '09], [Megías et al. '11]

$$(\sigma^{\mathcal{B}})_{AB} = \frac{1}{8\pi^2} \sum_C \mathrm{tr}\left(T_A\{T_B, H_C\}\right) \mu^C = \frac{1}{4\pi^2} \underbrace{d_{ABC}}_{\text{Anomaly Coeff.}} \mu^C \,.$$

Chiral Magnetic Conductivity induced by the Chiral Anomaly.

Kubo Formulas

Triangle Diagram



Energy Transport and Chiral Vortical Effect

[I. Amado, K. Landsteiner, F. Pena-Benitez, JHEP1105 '11]

- finite density: charge transport $\Longrightarrow$ energy transport

$$\delta T_{0i} = \mu \delta J_i \overset{\delta J_i = \delta\sigma^{\mathcal{B}} B_i}{=} \mu \delta \sigma^{\mathcal{B}} B_i \,.$$

- Energy flux sourced by magnetic fields

$$\sigma^{\mathcal{V}} = \frac{i}{2p_j} \sum_{i,k} \epsilon_{ijk} \langle J_i T_{0k} \rangle \Big|_{\omega=0} = \int \mu d\sigma^{\mathcal{B}} + const$$

- $T_{\mu\nu}$ sourced by metric

$$ds^2 = -(1-2\Phi)dt^2 + 2\vec{A}_g dt d\vec{x} + (1+2\Phi)d\vec{x}^2 \,,$$

- A_g "gravitomagnetic field" $\Longrightarrow$ chiral gravitomagnetic effect

$$\vec{J} = \sigma^{\mathcal{V}} \vec{B}_g \,, \qquad B_g^i = \epsilon^{ijk} \partial_j A_{g,k} \,.$$

- Chiral vortical effect: local fluid velocities

$$u^\mu = (1,0,0,0) \,, \qquad u_\mu = (-1, \vec{A}_g) \,,$$
$$J^i = \sigma^{\mathcal{V}} \epsilon^{ijk} \partial_j u_k \,, \qquad \sigma^{\mathcal{V}} \equiv \text{Vorticity coeff.}$$

Hydrodynamics of Relativistic Fluids

[Son,Surowka], [Eling,Neiman,Oz], [Erdmenger et al.], [Banerjee et al.], [Loganayagam], [Kharzeev, Yee], [Sadovyev et al.]

$$T^{\mu\nu} = \underbrace{(\epsilon + P)u^\mu u^\nu + P g^{\mu\nu}}_{\text{Ideal Hydro}} + \underbrace{\tau^{\mu\nu}}_{\text{Dissipative+Anomalous}} ,$$

$$J^\mu = \underbrace{n u^\mu}_{\text{Ideal Hydro}} + \underbrace{\nu^\mu}_{\text{Dissipative+Anomalous}} .$$

- Landau frame: $T^{0i} = (\epsilon + P)u^i$

$$\tau^{\mu\nu} = -\eta P^{\mu\alpha} P^{\nu\beta} \left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3} g_{\alpha\beta} \nabla^\lambda u_\lambda \right) - \zeta P^{\mu\nu} \nabla^\alpha u_\alpha + \cdots$$

$$\nu^\mu = -\sigma T P^{\mu\nu} \nabla_\nu \left(\frac{\mu}{T} \right) + \sigma E^\mu + \xi^{\mathcal{B}} B^\mu + \xi^{\mathcal{V}} \omega^\mu + \cdots$$

where $P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$, and vorticity: $\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\lambda} u_\nu \nabla_\rho u_\lambda$.

Hydrodynamics of Relativistic Fluids

- Laboratory rest frame:

$$\sigma^{\mathcal{B}} = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \langle J^a J^b \rangle |_{\omega=0} \, ,$$

$$\sigma^{\mathcal{V}} = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \langle J^a T^{0b} \rangle |_{\omega=0} \, .$$

- Landau frame:
  ξ's coefficients are different from σ's, but they are related to them:

$$\xi^{\mathcal{B}} = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \left(\left\langle J^a J^b \right\rangle - \frac{n}{\epsilon + P} \left\langle T^{0a} J^b \right\rangle \right) \Big|_{\omega=0} \, ,$$

$$\xi^{\mathcal{V}} = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{nkl} \left(\langle J^a T^{0b} \rangle - \frac{n}{\epsilon + P} \langle T^{0a} T^{0b} \rangle \right) \Big|_{\omega=0} \, .$$

Issues

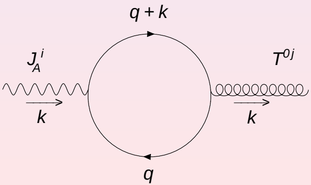
Field Theory Computation

- As before: general symmetry group

$$T^{0i} = \frac{i}{2}\sum_{f=1}^{N} \bar{\Psi}_f(\gamma^0\partial^i + \gamma^i\partial^0)\mathcal{P}_+\Psi^f \,.$$

- Chiral Vortical Conductivity: defined from the retarded correlation function of $J^i_A(x)$ and $T^{0j}(x')$, i.e.

$$G^{\mathcal{V}}_A(x-x') = \frac{i}{2}\epsilon_{ijn}\;\theta(t-t')\,\langle[J^i_A(x), T^{0j}(x')]\rangle \,.$$

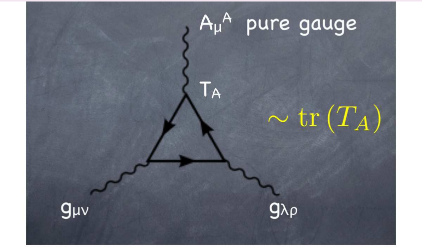


Field Theory Computation

- 1 loop calculation **[Landsteiner, Megías, Pena-Benitez, PRL107 '11]**:

$$(\sigma^{\mathcal{V}})_A = \lim_{k_n \to 0} \sum_{ij} \epsilon_{ijn} \frac{-i}{2k_n} \langle J_A^i T^{0j} \rangle |_{\omega=0} = \frac{1}{8\pi^2} \sum_{f=1}^{N} (T_A)^f{}_f \left[(\mu^f)^2 + \frac{\pi^2}{3} T^2 \right]$$

$$= \frac{1}{8\pi^2} \sum_{B,C} d_{ABC} \mu^B \mu^C + \underbrace{\frac{T^2}{24} \mathrm{tr}\,(T_A)}_{\text{Gravitational Anomaly}}$$



Field Theory Computation

- Anomaly:

$$\nabla_\mu J^\mu_A = \epsilon^{\mu\nu\rho\lambda}\left(\frac{d_{ABC}}{32\pi^2} F^B_{\mu\nu} F^C_{\rho\lambda} + \frac{b_A}{768\pi^2} R^\alpha{}_{\beta\mu\nu} R^\beta{}_{\alpha\rho\lambda}\right) ,$$

$$d_{ABC} = \frac{1}{2}[\mathrm{tr}(T_A\{T_B, T_C\})_R - \mathrm{tr}(T_A\{T_B, T_C\})_L] ,$$

$$b_A = \mathrm{tr}(T_A)_R - \mathrm{tr}(T_A)_L .$$

- Special case: vector and axial current for one Dirac fermion
  - Vector vortical conductivity $\sigma^{\mathcal{V}}_V = \frac{\mu\mu_A}{2\pi^2}$ (no grav. anom.)
  - Axial vortical conductivity

$$\sigma^{\mathcal{V}}_A = \frac{\mu^2 + \mu_A^2}{4\pi^2} + \frac{T^2}{12}$$

See also [Vilenkin '80].

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Holography

[K. Landsteiner, E. Megías, L. Melgar, F. Pena-Benitez, JHEP 1109:121(2011)]

- Mixed gauge-gravitational Chern Simons term

$$S = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\Big[R + 2\Lambda - \frac{1}{4}F_{MN}F^{MN}$$
$$+\epsilon^{MNPQR}A_M\left(\frac{\kappa}{3}F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)\Big] + S_{GH} + S_{CSK}$$

- To reproduce gravitational anomaly at general hypersurface (not only $r\to\infty$)

$$S_{CSK} = -\frac{1}{2\pi G}\int_\partial d^4x\sqrt{-h}\lambda n_M\epsilon^{MNPQR}A_N K_{PL}D_Q K^L_R\,.$$

- Equations of motion

$$G_{MN} - \Lambda g_{MN} = \frac{1}{2}F_{ML}F_N{}^L - \frac{1}{8}F^2 g_{MN} + 2\lambda\epsilon_{LPQR(M}\nabla_B\left(F^{PL}R^B{}_{N)}{}^{QR}\right),$$
$$\nabla_N F^{NM} = -\epsilon^{MNPQR}\left(\kappa F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right).$$

Holography

- Current:

$$J^\mu = -\frac{\sqrt{-\gamma}}{16\pi G}\left[F^{r\mu} + \frac{4}{3}\kappa\epsilon^{\mu\nu\rho\lambda}A_\nu F_{\rho\lambda}\right]_\partial .$$

- On-shell we recover the anomaly

$$D_\mu J^\mu = -\frac{1}{16\pi G}\epsilon^{\mu\nu\rho\lambda}\left(\kappa F_{\mu\nu}F_{\rho\lambda} + \lambda R^{\alpha}_{(4)\ \beta\mu\nu}R^{\beta}_{(4)\ \alpha\rho\lambda}\right)$$

$$\Longrightarrow -\frac{\kappa}{16\pi G} = \frac{1}{32\pi^2}\,, \quad -\frac{\lambda}{16\pi G} = \frac{1}{768\pi^2}\,.$$

- Energy-momentum tensor

$$T^{\mu\nu} = \frac{\sqrt{-\gamma}}{8\pi G}\left[K^{\mu\nu}_{(4)} - K_{(4)}\gamma^{\mu\nu} + 4\lambda\epsilon^{(\mu\rho\lambda\alpha}\left(\frac{1}{2}F_{\rho\lambda}R^{\nu)}_{\alpha} + D_\beta(A_\rho R^{\beta\nu)}{}_{\lambda\alpha})\right)\right]_\partial .$$

- Energy-momentum (non)-conservation relation (see also **[Neiman,Oz '11]**)

$$D_\nu T^{\nu\mu} = F^{\mu\nu}J_\nu - A^\mu D_\alpha J^\alpha\,.$$

Holographic Renormalization

[Martelli, Mueck '03], [Papadimitriou, Skenderis '04], [Clark,Love,Veldhuis '10], [Landsteiner, Megías, Melgar, Pena-Benitez '11]

- Divergent terms in the action are (eigenvalues of the dilatation operator):

$$S_{div} = S_{(0)} + S_{(2)} + \tilde{S}_{(4)} \, .$$

- The counterterm for the on-shell action is:

$$\begin{aligned} S_{ct} &= -\frac{(d-1)}{8\pi G} \int_\partial d^4x \sqrt{-\gamma} \Big[1 + \frac{1}{(d-2)} P \\ &\quad - \frac{1}{4(d-1)} \left(P^\mu_\nu P^\nu_\mu - P^2 - \frac{1}{4} F_{(0)\,\mu\nu} F_{(0)}{}^{\mu\nu} \right) \log e^{-2r} \Big] \, , \end{aligned}$$

where $P = \frac{R_{(4)}}{2(d-1)}$, $P^\mu_\nu = \frac{1}{2} \left[R^\mu_{(4)\nu} - P \delta^\mu_\nu \right]$.

Gauge-gravitational Chern Simons term does not induce new divergences, and the renormalization is not modified by it ☺

(See also Clark, Love & Veldhuis '10).

Transport coefficients

- Background: charged AdS black hole

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f(r)}dr^2 , \quad f(r) = 1 - \frac{ML^2}{r^4} + \frac{Q^2L^2}{r^6} ,$$
$$A_{(0)} = \phi(r)dt = \left(\alpha - \frac{\mu r_h^2}{r^2}\right) dt .$$

- Mass, Charge and Temperature of the Reissner-Nordström black hole

$$M = \frac{r_h^4}{L^2} + \frac{Q^2}{r_h^2} , \quad Q = \frac{\mu r_h^2}{\sqrt{3}} , \quad T = \frac{r_h^2}{4\pi L^2} f'(r_h) = \frac{(2 r_h^2 M - 3 Q^2)}{2\pi r_h^5}$$

- Fluctuations in shear sector

$$A_M = A_M^{(0)} + \epsilon\, a_M , \qquad g_{MN} = g_{MN}^{(0)} + \epsilon\, h_{MN} .$$

Gauge field: $a_x , a_z .$ Metric: $h_t^x , h_t^z , h_y^x , h_y^z .$

Transport coefficients

[Kaminski, Landsteiner, Mas, Shock, Tarrio, JHEP02 '10]

- Second order action on-shell (expansion $\mathcal{O}(\epsilon^2)$)

$$\delta S^{(2)}_{ren} = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \{\Phi^I_{-k} \mathcal{A}_{IJ} \Phi'^J_k + \Phi^I_{-k} \mathcal{B}_{IJ} \Phi^J_k\}\Big|_{r\to\infty} ,$$

where $\Phi^I_k \equiv \left(\frac{a_x}{\mu}, h^x_t, \frac{a_z}{\mu}, h^z_t\right)$, and

$$\mathcal{A} = \frac{r_h^4}{16\pi G L^5} \mathrm{Diag}\left(-3af, \frac{1}{u}, -3af, \frac{1}{u}\right) , \quad u = \frac{r_h^2}{r^2} , \quad a = \frac{\mu^2 L^2}{3 r_h^2} ,$$

$$\mathcal{B}_{AdS+\partial} = \frac{r_h^4}{16\pi G L^5} \begin{pmatrix} 0 & -3a & \frac{4\kappa i k \mu^2 \phi L^5}{3 r_h^4} & 0 \\ 0 & -\frac{3}{u^2} & 0 & 0 \\ \frac{-4\kappa i k \mu^2 \phi L^5}{3 r_h^4} & 0 & 0 & -3a \\ 0 & 0 & 0 & -\frac{3}{u^2} \end{pmatrix} ,$$

$$\mathcal{B}_{CT} = \frac{r_h^4}{16\pi G L^5} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{3}{u^2\sqrt{f}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{u^2\sqrt{f}} \end{pmatrix} , \qquad \mathcal{B} = \mathcal{B}_{AdS+\partial} + \mathcal{B}_{CT} .$$

Transport coefficients

- Bulk solutions ($\varphi_k^J \equiv$ boundary values)

$$\Phi_k^I(r) = F^I{}_J(k,r)\,\varphi_k^J\,, \qquad F^I{}_J(k,r) \stackrel{r\to\infty}{\rightarrow} 1\,.$$

- Retarded Green functions

[Son,Starinets '02], [Herzog, Son '03], [Kaminski et al '10], [Amado et al '09]

$$G_{IJ}(k) = -2\lim_{r\to\infty}\left(\mathcal{A}_{IM}(F^M{}_J(k,r))' + \mathcal{B}_{IJ}\right)\,.$$

- Correlators are

$$\langle JJ\rangle = -ip_z\left(\frac{\kappa}{2\pi G}\mu - \frac{\kappa}{6\pi G}\alpha\right) \Longrightarrow \sigma^{\mathcal{B}} = \frac{\mu}{4\pi^2}$$

$$\langle JT\rangle = -ip_z\left(\frac{\kappa}{4\pi G}\mu^2 + \frac{2\pi\lambda}{G}T^2\right) \Longrightarrow \sigma^{\mathcal{V}} = \frac{\mu^2}{8\pi^2} + \frac{T^2}{24}$$

$$\langle TT\rangle = -ip_z\left(\frac{\kappa}{6\pi G}\mu^3 + \frac{4\pi\lambda}{G}\mu T^2\right) \Longrightarrow \sigma^{\epsilon,\mathcal{V}} = \frac{\mu^3}{12\pi^2} + \frac{\mu T^2}{12}$$

Coefficients consistent with weak coupling.
No T^3 terms! (CPT invariance)

See also [Neiman,Oz], [Loganayagam].

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Fluid/Gravity Correspondence

[Erdmenger et al.], [Bhattacharyya et al.], [Banerjee et al.].

- Boosted black branes:

$$ds^2 = -r^2 f(r) u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu - 2u_\mu dx^\mu dr$$

where

$$f(r) = 1 - \frac{M}{r^4} + \frac{Q^2}{r^6}\,, \qquad P_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu\,,$$

$$A_r = 0\,, \qquad A_\mu = -\frac{\sqrt{3}Q}{r^2} u_\mu\,.$$

- $u_\mu(x), M(x), Q(x)$ are slowly varying. This means $\partial X \ll X/\ell_{mfp}$
- Then one can compute the fields in a derivative expansion:

$$\begin{aligned} g_{AB} &= g_{AB}^{(0)} + g_{AB}^{(1)} + g_{AB}^{(2)} + \cdots \\ A_M &= A_M^{(0)} + A_M^{(1)} + A_M^{(2)} + \cdots \end{aligned}$$

Fluid/Gravity Correspondence

- Decompose the fields into scalar, vector and tensor sectors:

$$\begin{aligned} ds^2 &= r^2 k(r) u_\mu u_\nu dx^\mu dx^\nu + r^2 h(r) P_{\mu\nu} dx^\mu dx^\nu + r^2 \pi_{\mu\nu}(r) dx^\mu dx^\nu \\ &\quad + r^2 j_\sigma(r)(P^\sigma_\mu u_\nu + P^\sigma_\nu u_\mu) dx^\mu dx^\nu - 2S(r) u_\mu dx^\mu dr \,, \\ A_r &= 0 \,, \qquad A_\mu = a_\nu(r) P^\nu_\mu + c(r) u_\mu \,. \end{aligned}$$

- Compute $k(r), h(r), \ldots$ in a derivative expansion of $Q(x)$, $M(x)$ and $u_\mu(x)$.
- Zeroth order is trivial:

$$k^{(0)}(r) = -f(r) \,, \quad h^{(0)}(r) = 1 \,, \quad S^{(0)}(r) = 1 \,, \quad c^{(0)}(r) = -\frac{\sqrt{3}Q}{r^2}$$

and $= 0$ the rest.

Fluid/Gravity Correspondence

Boundary conditions when solving Einstein-Maxwell equations:

1. Choice (A), (B) or (C).
2. $a_\mu =$ finite at the horizon.
3. $j_\mu = 0$ at the boundary.
4. $a_\mu = 0$ at the boundary.

- Choice (A):
  1. $j_\mu^{(\bar{4},1)} = 0$ in near boundary expansion $j_\mu^{(1)} \simeq \cdots + \frac{j_\mu^{(\bar{4},1)}}{r^4} + \cdots$

  It is the Landau frame condition.

  $\Longrightarrow$ Transport coefficients in Landau frame.
- Choice (B):
  1. $j_\mu = 0$ at the horizon.

  $\Longrightarrow$ Transport coefficients in Laboratory rest frame.
- Choice (C):
  1. $a_\mu^{(\bar{2},1)} = 0$ in the near boundary expansion $a_\mu^{(1)} \simeq \cdots + \frac{a_\mu^{(\bar{2},1)}}{r^2} + \cdots$

  $\Longrightarrow$ Transport coefficients in Eckart frame.

Fluid/Gravity Correspondence

- In Laboratory rest frame (Choice (B)):

$$a_\mu^{(1)}(r) = \cdots - \left[\frac{6Q^2}{r_+^4} \frac{\kappa}{r^2} + \frac{4(Q^2 - 2r_+^6)^2}{r_+^{10}} \frac{\lambda}{r^2} + \mathcal{O}(r^{-6}) \right] \omega_\mu + \ldots$$

- Current:

$$\langle J_\mu \rangle = \frac{1}{\sqrt{-g^{(\bar{0})}}} \frac{\delta}{\delta A^{\mu\,(\bar{0})}} S_{\rm ren}[A_\mu^{(\bar{0})}, g_{\mu\nu}^{(\bar{0})}] \quad = \quad -\frac{1}{8\pi G} a_\mu^{(\bar{2})}$$

$$= \quad \cdots + \sigma^{\mathcal{B}} B_\mu + \sigma^{\mathcal{V}} \omega_\mu + \ldots$$

- Transport coefficients in Laboratory rest frame:

$$\sigma^{\mathcal{B}} = \frac{\mu}{4\pi^2},$$

$$\sigma^{\mathcal{V}} = \frac{\mu^2}{8\pi^2} + \frac{T^2}{24}$$

Conclusions

- We have studied the effects of external magnetic fields and vortices in a relativistic fluid.
- Anomalies $\Longrightarrow$ parity violating transport.
- We have derived Kubo formulas.
- Surprise: mixed gauge-gravitational anomaly contributes!!!
- Holography with gauge-gravitational Chern Simons term. Two methods:
  - Kubo Formulas.
  - Fluid/Gravity Correspondence.
- (Non)-renormalization of anomalous conductivities.
- Observable effects in heavy ion physics and cosmology?:
  - Enhanced production of high spin hadrons, especially Ω^- baryons, due to chiral separation effect **[Keren-Zur, Oz '10]**.
  - Lepton number separation in the early universe due to gravitational anomaly **[Alexander, Peskin, Sheikh-Jabbari '06]**.

[Kharzeev, Son '11], [Kharzeev, Yee '11].

Backup slides

Chemical Potentials for Anomalous Symmetries

Two formalisms to chemical potentials **[T.Evans '95]**:

Formalism	Hamiltonian	Boundary conditions
(A)	$H - \mu Q$	$\Psi(\tau) = -\Psi(\tau - \beta)$
(B)	H	$\Psi(\tau) = -e^{\beta\mu}\Psi(\tau - \beta)$

- If Q is non anomalous $\Longrightarrow$ (A) and (B) are connected by a large gauge transformation: $A_0 \to A_0 + \partial_0 \chi$ with $\chi = -\mu t$
- If Q is anomalous, the action transforms (in formalism (B)):

$$S[A+\partial\chi] = S[A] + \int d^4x\, \chi \epsilon^{\mu\nu\rho\lambda} \left(C_1 F_{\mu\nu} F_{\rho\lambda} + C_2 R^{\alpha}{}_{\beta\mu\nu} R^{\beta}{}_{\alpha\rho\lambda} \right) .$$

The gauge transformation leads to an inequilavent theory!!!

Chemical Potentials for Anomalous Symmetries

- Introduce a non-dynamical “axion” field:

$$S_{\rm tot}[A,\theta] = S[A] + \underbrace{\int d^4x\, \theta \epsilon^{\mu\nu\rho\lambda} \left(C_1 F_{\mu\nu}F_{\rho\lambda} + C_2 R^{\alpha}{}_{\beta\mu\nu} R^{\beta}{}_{\alpha\rho\lambda}\right)}_{\equiv S_\theta[A,\theta]} .$$

- Demand the axion transforms as $\theta \to \theta - \chi \Longrightarrow S_{tot}[A,\theta]$ is gauge invariant.
- Under a gauge transformation $\chi = \mu t$:

$$\begin{array}{rcl} \text{Formalism (B)} & \longrightarrow & \text{Formalism (A)} \\ A_0 = 0 & \longrightarrow & A_0 + \partial_0 \chi = \mu \\ \theta = 0 & \longrightarrow & \theta - \chi = -\mu t \\ H & \longrightarrow & H - \mu Q - 4\mu C_1 \int d^3x\, \epsilon^{0ijk} A_i \partial_j A_k \end{array}$$

This Hamiltonian is similar to the one advocated by **[Rubakov ’10]**.

Chemical Potentials for Anomalous Symmetries

Formalism	Hamiltonian	Boundary conditions
(A)	$H - \mu\,(Q + \mathcal{A})$	$\Psi(\tau) = -\Psi(\tau - \beta)$
(B)	H	$\Psi(\tau) = -e^{\beta\mu}\Psi(\tau - \beta)$
(G)	$H - \alpha\,(Q + \mathcal{A})$	$\Psi(\tau) = -e^{\beta(\mu-\alpha)}\Psi(\tau - \beta)$

- AdS Reissner-Nordström black-brane solution of the Holographic model:

$$A_0 = \phi(r)dt = \left(\alpha - \frac{\mu r_h^2}{r^2}\right) dt\,.$$

  - Choice (A): $\alpha = \mu \Longrightarrow A_0|_{r=r_h} = 0$
  - Choice (B): $\alpha = 0$.
- Choice (A) corresponds to Formalism (A), choice (B) to Form (B).
- (A) and (B) are physically equivalent, but we need to take into account the pure boundary action $S_\theta[A, \theta]$.
- θ does not extend into the bulk $\Longrightarrow$ non dynamical.
- Ex: Choice (A) with no S_θ gives a vanishing chiral magnetic conductivity. But with S_θ it gives the correct result ☺